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REVIEW OF SELECTED TOPICS IN HQET†

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Abstract

A few topics on the expansion in heavy quark mass are discussed. The theoretical framework is the Wilson Operator Product Expansion rather than the Heavy Quark Effective Theory.

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1 Introduction

The notion of Heavy Quark Effective Theory (HQET) was introduced by Eichten, Hill[1] and Georgi[2]. Then HQET and its applications were actively developed by many authors (see a recent review[3]). The theory is based on the smallness of the parameter Λ_{QCD}/m_Q, i.e. the ratio of characteristic hadronic scale to the scale given by heavy quark mass m_Q, and on the existence of the limit $m_Q \to \infty$.

The recent development is concentrated mostly on preasymtotic effects, i.e. on the study of nonpertubative corrections $(\Lambda_{QCD}/m_Q)^n$. The main theoretical tool is the Operator Product Expansion (OPE) introduced by Wilson[4] which allows a separation of short and large distances. What is a relation between HQET and OPE? In a sense HQET is just a particular application of OPE. However, the way the HQET was implemented is not entirely consistent with the OPE. One of basic quantities in HQET is the pole quark mass. The existence of infrared contributions in the pole quark mass leads to problems when $(1/m_Q)^n$ correction are taken into account. At this level the standard HQET does not exist as a quantum field theory[5, 6]. For this reason I prefer to refer to the OPE rather to the HQET.

There is a clear analogy between the use of OPE for heavy flavour physics and classical applications of OPE to e^+e^- annihilation into hadrons and to deep inelastic scattering. First, heavy flavour states can be viewed as ground states of light flavours but in the presence of almost static gluon field produced by a heavy quark (as different from vacuum or nucleon states). Second, the analogy between short distance probes is the analogy between, say, the total cross section of hadron production in e^+e^- annihilation versus the total semileptonic widths of heavy flavours. The heavy quark mass m_Q plays the role similar to the total energy W in e^+e^- collisions defining the scale for perturbative and nonperturbative corrections.

Let me finish this short introduction presenting the partial list of topics where a theoretical understanding was strongly advanced during recent years:

• Corrections $1/m_Q^2$ to inclusive widths. Spectra near end-points – QCD description of the "Fermi motion" of heavy quark.

• Pole mass: infrared renormalons and m_Q^{pole} uncertainty, the necessity of normalization point, the correct OPE construction.

• Sum rules for heavy flavour transitions. New sum rules, $(1/m_Q)^n$ correction the known ones, lower bound for the average kinetic energy of heavy quark.

• Extraction of $|V_{cb}|$ from exclusive ($B \to D^* l\nu$) and inclusive ($B \to X_c l\nu$) processes.

• Status of semileptonic branching ratio.

2 Total Widths

As an example of theoretical predictions let us present a result for the total width of semileptonic decay $B \to X_u l\nu$,

$$\Gamma(B \to X_u l\nu) = \frac{G_F^2 m_b^5 |V_{ub}|^2}{192\pi^3}\left[1 - \frac{\mu_\pi^2}{2m_b^2} - \frac{3\mu_G^2}{2m_b^2}\right] . \tag{1}$$

Here μ_G^2 and μ_π^2 are defined as

$$\mu_G^2 = \frac{1}{2m_b}\langle B|\bar{b}\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle, \quad \mu_\pi^2 = \frac{1}{2m_b}\langle B|\bar{b}(i\vec{D})^2 b|B\rangle. \tag{2}$$

The numerical value of μ_G^2 is known from the hyperfine splitting of B^* and B mesons:

$$\mu_G^2 = \frac{3}{4}(M_{B^*}^2 - M_B^2) \approx 0.36\,\mathrm{GeV}^2 . \tag{3}$$

Note that for baryons (besides Ω_Q) $\mu_G^2 = 0$. As for μ_π^2 it is not yet extracted from experimental data, there is only the number which follows from QCD sum rules[7], $\mu_\pi^2 = 0.5 \pm 0.1$.

Numerically power corrections diminish the width determined in equation 1 by about 3%. In a similar fashion corrections which are parametrically $1/m_b^2$ to other semileptonic and nonleptonic widths are expressed via the same μ_G^2, μ_π^2. The absence of corrections of the first power in $1/m_Q$ is specific for QCD as it was pointed out first in paper[8] (with some reservations about overall normalization). Explicitly terms $1/m_Q^2$ were calculated in[9] where the absence of the $1/m_Q$ corrections was stated for the normalization as well.

Corrections $1/m_Q^2$ are "spectator blind", i.e. do not depend on the flavour of light quark, say, in a heavy meson. The dependence shows up at the level of $1/m_Q^3$ terms[10] which numerically are comparable with $1/m_Q^2$ terms. The overall fit for lifetimes of charm and beauty hadrons looks satisfactory[11] with Λ_b as an exception. Experimentally $\tau(\Lambda_b)/\tau(B_d) = 0.76 \pm 0.06$ with 0.9 as a preferable theoretical number.

3 End-point Spectra

Julia Ricciardi discussed in her talk at this Conference recent papers[12, 13] devoted to analysis of photon spectrum in $B \to X_s\gamma$ inclusive decay. I would like to add some comments on the topic.

It was realized long ago that the window $m_b/2 < E_\gamma < M_B/2$ in the photon spectra which is empty on the parton level is filled up only due to the nonperturbative effect of the heavy quark motion, and phenomenological models accounting for the effect were suggested[14].

What is the QCD (i.e. model independent one) description? It was worked out in papers[15]–[18]. Near end-points the energy release for the light quark system at the final state is not of the order of m_b which is much larger than the QCD scale Λ_{QCD} but of the order of $\bar{\Lambda} = M_B - m_b$, which is $\sim \Lambda_{QCD}$. Thus operators of high dimension in the OPE are not suppressed by small coefficients ($\propto 1/m_Q^n$) and the summation is needed. This summation is a natural generalization of the OPE procedure for deep inelastic processes.

The quantity which substitutes Q^2 is K^2,

$$K^2 = -k^2 = -(m_b v_\mu - q)^2 = 2m_b(E_\gamma - \frac{m_b}{2}), \tag{4}$$

where q is the lepton pair momentum and $v_\mu = p_\mu^B/M_B$ is 4-velocity of B meson. At the end-point region $K^2 \sim m_b\bar{\Lambda}$ which is much larger than $\bar{\Lambda}^2$ but still much less than m_b^2. Perturbative corrections are governed by $\alpha_s(K^2)$. Nonperturbative terms are given by the following sum:

$$\langle B|\bar{b}\frac{2}{K^2}\sum_{n=0}^{\infty}\left(\frac{2k\pi}{K^2}\right)^n b|B\rangle, \tag{5}$$

where $\pi_\mu = iD_\mu$. All terms in this expansion are of the same order at the end-point region and present moments of distribution function $F(x)$. The scaling variable x (an analog of Bjorken x) is defined as

$$x = \frac{K^2}{2\bar{\Lambda}(kv)} \approx (q_0 + |\vec{q}| - m_b)/\bar{\Lambda}\ .$$

From equation 5 first few moments of $F(x)$ are as follows:

$$\int dxF(x) = 1,\ \int dxxF(x) = 0,\ \int dxx^2F(x) = \frac{\mu_\pi^2}{3\bar{\Lambda}^2},$$

$$\int dxx^3F(x) = \frac{1}{6\bar{\Lambda}^3}\frac{1}{2M_b}\langle B|\bar{b}\gamma_0 b\cdot g_s\sum_q \bar{q}\gamma_0 q|B\rangle. \tag{6}$$

The distribution function $F(x)$ is universal in the sense that it defines spectra of different processes where final quarks are relativistic. In particular, it is the case for $B\to X_s\gamma$ and $B\to X_u l\bar{\nu}$ decays,

$$\frac{d\Gamma(B\to X_s\gamma)}{dE_\gamma} = \Gamma_0^{s\gamma}\,\frac{2F(x)}{\bar{\Lambda}};$$

$$\frac{d\Gamma(B\to X_u l\bar{\nu})}{dE_l dq^2 dq_0} = \frac{\Gamma_0^{ul\nu}}{m_b^5}\,\frac{2F(x)}{\bar{\Lambda}}\frac{24(q_0-E_l)(2m_bE_l-q^2)}{m_b-q_0}\ . \tag{7}$$

Then the lepton spectrum near the end point is proportional to

$$\int_{(2E_l-m_b)/\bar{\Lambda}}^1 dxF(x)\ .$$

Near the end points perturbative corrections are enhanced and the summations of leading (double) logs as well as subleading is required. The resulting perturbative kernel should be convoluted with the distributions given in equation 7. The most advanced realization of this program given in the paper[13].

Let me emphasize that an absolutely different distribution function appears when the final quark is slow. The realistic case of $B \to X_c l\bar{\nu}$ decay is a mixed one but the c quark is predominantly nonrelativistic over phase space in this decay.

4 Irrelevance of Pole Mass and Infrared Renormalons

Let us start from the one-loop correction to the heavy quark mass m_Q. This correction is described by a simple diagram with Coulomb quanta exchange along fermionic line. The integration over the gluon momentum k is limited by μ_0 from above and by μ from below, $\mu_0 \ll m_Q$. The result looks as follows:

$$m_Q(\mu) = m_Q(\mu_0) + \frac{2\pi}{3}\frac{\alpha_s}{\pi}(\mu_0 - \mu). \tag{8}$$

This equation presents running of m_Q in the range of $\mu \ll m_Q$ and has a simple meaning of accounting for the Coulomb energy $2\alpha_s/3r_0$. The equation 8 reflects the infrared stability of the mass – the limit $\mu \to 0$ does exist and corresponds to the pole mass.

However, accounting for the running of α_s makes the result for the pole mass uncertain[5, 6]. Indeed, let us substitute $\alpha_s(k^2)/k^2$ for the gluon propagator in the simplest diagram discussed above. Then higher order terms appear in $m_Q(\mu)$:

$$m_Q(\mu) - m_Q(\mu_0) = \frac{4\alpha_s}{3\pi}\mu \sum 2^n n! \left(\frac{b\alpha_s}{4\pi}\right)^n, \tag{9}$$

where b is the one-loop β function coefficient. The factorial divergence of this expansion reflects an appearance of infrared renormalons in the problem. One can try to sum up the expansion 9 but different prescriptions lead to different results with an uncertainty $\sim \Lambda_{QCD}$.

The relative uncertainty Λ_{QCD}/m_Q seems to be a clear contradiction to the statement discussed above about the absence of $1/m_Q$ corrections to inclusive widths (see the equation 1). The parodox is simply resolved since inclusive widths are defined by short distances $\sim 1/m_Q$ and masses should be correspondingly taken at deeply Eucledian distances as well. Only in terms of these masses does the statement that there is no $1/m_Q$ corrections make sense.

Of course, in any given order in α_s it is possible to formulate results in terms of the pole mass. It means, however, the factorial behaviour of coefficients in perturbation theory and

the renormalon uncertainty. On the other hand, the consistent use of masses normalized at the relevant distances shows that both the pole mass and infrared renormalons in the coefficients are physically irrelevant.

It is just the place where the standard HQET occurs to be inconsistent because the notion of normalization point was not introduced there for power corrections. There was a hot discussion in the literature (see e.g. paper[19], see also paper[20] where the lattice is used for nonperturbative definition of the pole mass) concerning a consistency of HQET. I do not think that the necessity of explicit introduction of normalization point can be avoided.

5 Sum Rules

In semileptonic $b \to c$ transitions the c quark can be treated also as a heavy one. This leads to some number of sum rules for the moments of spectral distributions such as:

$$I_n(\vec{q}) = \frac{1}{2}\left(\delta_{kl} - \frac{q_k q_l}{\vec{q}^2}\right) \sum_i \epsilon_i^n \langle B|j_k|i\rangle\langle i|j_l|B\rangle \tag{10}$$

where $\epsilon_i = \sqrt{M_i^2 + \vec{q}^2} - \sqrt{M_0^2 + \vec{q}^2}$ is an excitation energy of i-th state moving with momentum $(-\vec{q})$. The current j_k here is the axial current $j_k = \bar{c}\gamma_k\gamma_5 b$. Referring to[21] for details let us discuss few sum rules arising in slow velocity[22] (SV) limit.

$$I_0(\vec{q}=0) = 1 - \frac{\mu_G^2}{3m_c^2} - \frac{\mu_\pi^2 - \mu_G^2}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}\right). \tag{11}$$

The radiative corrections are omitted here. The unity in this sum rule corresponds to Bjorken sum rule[23] at zero recoil, and the next terms are the $1/m_Q^2$ corrections. Numerically the term $\mu_G^2/3m_c^2$ decreases the total normalization by about 7%.

The $1/m_c^2$ corrections are much larger for the derivative of I_0,

$$m_c^2 \frac{dI_0}{d\vec{q}^2}|_{\vec{q}=0} = -\frac{1}{4}\left[1 - 3\frac{\mu_G^2}{m_c^2} - (\mu_\pi^2 - \mu_G^2)\left(\frac{5}{2m_c^2} + \frac{1}{2m_b^2} + \frac{1}{m_c m_b}\right)\right] . \tag{12}$$

Corrections now are about 50%!

In this and in the previous sum rule terms with $\mu_\pi^2 - \mu_G^2$ only enhance the effect – this difference is positive as a consequence of certain some rule. It gives a lower bound for $\mu_\pi^2 \geq \mu_G^2 = 0.36\,\mathrm{GeV}^2$. The uncertainty in this bound due to radiatvie corrections was analyzed[21] and numerically is about $0.1\,\mathrm{GeV}^2$.

The derivative of the first moment looks as follows:

$$m_c^2 \frac{dI_1}{d\vec{q}^2}|_{\vec{q}=0} = \frac{1}{2}\left[\bar{\Lambda} - \frac{\bar{\Lambda}^2}{m_c} - \frac{4\mu_G^2}{3m_c} - \frac{\mu_\pi^2 - \mu_G^2}{2m_c}\left(\frac{7}{6} + \frac{m_c}{3m_b}\right)\right]. \tag{13}$$

The first term $\bar{\Lambda}$ presents Voloshin sum rule[24], the next ones are the $1/m_Q$ corrections. The term $-4\mu_G^2/3m_c \approx -0.3\,\mathrm{GeV}$.

The derivative of the second moment gives the sum rule[25] for μ_π^2:

$$m_c^2 \frac{dI_2}{d\vec{q}^2}|_{\vec{q}=0} = \frac{\mu_\pi^2}{3}\ . \tag{14}$$

We see from examples above that c quark is not heavy enough to guarantee a smallness of power corrections.

6 Extraction of $|V_{cb}|$

There are two sources for extraction of the value of $|V_{cb}|$: the rate of exclusive decay $B \to D^* l\nu$ in the zero-recoil limit and the inclusive total width of semileptonic decay $B \to X_c l\nu$. The main problem is a reliable estimate of theoretical uncertainty. Two years ago a common belief was that the exclusive decay rate gives a more accurate extraction. Now it is known to be not the case – the inclusive width leads to the better accuracy.

What has been changed? First it was a demonstration[26] of relatively large $1/m_c^2$ corrections (see the equation 11 above). The left hand side of this equation is the sum over probabilities of transitions with $B \to D^*$ as a leading one. The main uncertainty comes from the conribution of excited states into this sum what gives additional $1/m_c^2$ corrections. The amplitude of $B \to D^*$ transition can be estimated as

$$F_{B\to D^*} \approx 0.9 \pm 0.035$$

while in 1993 it was[27] 1.00 ± 0.04.

Second, regarding the inclusive width it was noticed[26] that its dependence on the value of m_b is not so strong as it looks like from the m_b^5 factor – it depends the most strongly on $m_b - m_c$ which is defined by $M_B - M_D$. The uncertainty in the value of μ_π^2 becomes important but it can be improved when μ_π^2 will be extracted directly from the data. The result for $|V_{cb}|$ extracted from the inclusive width is (see[28, 29]):

$$|V_{cb}| = 0.408 \left[\frac{Br(B \to X_c l\nu)}{0.105}\right]^{1/2} \left[\frac{1.6\,ps}{\tau_B}\right]^{1/2} (1 \pm 0.03). \tag{15}$$

7 Semileptonic Branching Ratio

The theoretical understanding of this branching ratio is still lacking. Experimentally[31] it is $(10.43 \pm 0.24)\%$, the theoretical situation over time is presented in the following table.

Table 1: Semileptonic Branching Ratio (in %)

$\alpha_s(m_Z)$	Parton model [30] ('91)	HQE [32] ('94)	BBBG [33] ('94)
0.110	13.3	13.2	12.3
0.117	13.0	12.8	11.8
0.124	12.5	12.3	11.3

The second column gives results of leading and next-to-leading log summation of radiative corrections[30] for different values of α_s. The third column accounts for power corrections[32]. The last column from the paper[33] which gives the lowest values for the branching ratio accounts for two recent developments – first, the enhancement of nonleptonic $b \to \bar{c}cd$ transition by radiative corrections[34, 33], second, for the finite value of m_c/m_b in the radiative loops[33].

Although the theoretical branching ratio went down the problem does not seem resolved. The point is that the prediction[33] for the yield of c quarks in B decays is too high, $\langle n_c \rangle = 1.28 \pm 0.08$. The experimental value is $\langle n_c \rangle = 1.129 \pm 0.046$ (see the talk by Browder at this Conference).

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